An Approach for Determining Angle of Rotation of a Gray Image Using Weighted Statistical Regression

Joydev Hazra, Aditi Roy Chowdhury, Paramartha Dutta

Abstract— This article presents a computationally efficient method to estimate two dimensional in place rotation. This method is based on the estimation of weighted linear regression lines of the reference and the sense image pair. Angle between these two regression lines can identify the rotational angle between the reference and sense images. This fully unsupervised technique offers convincing result for wide range of images, without any need for setting control points. Another important contribution is that in composite rotation this method can effectively identify cumulative angle of rotation. The experiments show that the proposed method is robust and can be applied on various kind of image applications. Comparison with existing methods suggests the supremacy of the present one over its existing competitors.

Index Terms— Axis of symmetry, Composite Rotation, Hessian Matrix, Image Rotation , Index of symmetry, Weighted Centroid, Weighted Linear Regression Line.

1 Introduction

IMAGE processing has a huge research spectrum. Since several years, a lot of studies figure for image understanding and image analysis. Some research areas in 2D image analysis are multi-modality fusion [1], image coding, enhancement, compression, segmentation [2], [3], texture analysis, visual inspection, recognition and registration [4],[5],[6] etc. Image matching and image registration take an important role in computer vision, image sensing, image/video processing, etc. Transformation estimation is an important step of image matching.

1.1 Review

Over the years, research on image registration has offered a lot of methods. Typical examples include methods like image correlation functions, principal axis method, Fourier transform based methods, image feature based methods [7] and so on. Rotation detection is an important part in image matching.

Fourier-Mellin Invariant property is widely used in transformation detection. The Fourier transform has certain properties under rotation,scale,translation transformations that make it useful for rotation estimation. Rotating an image in the spatial domain by angle θ is equivalent to rotating the magnitude of its Fourier transform by $-\theta$ [8]. Scale and translation-invariant domain are used to recover rotation.

Log-polar method is one of the technique to estimate rotation [9]. Rotation invariance property of log- polar mapping is used to compute rotational angle. Different projection techniques are used on sense and reference images and estimation of rotation becomes the problem of estimating the shift between two one-dimensional signals.

In [10] the authors use the pseudo-polar transform to achieve some level of improved approximations of the polar and log-polar Fourier transforms of an image. Here rotations are reduced to translations which are estimated using phase correlation. 1D FFT operations are used to make it much faster than 2D FFT.

PCA (Principal Component Analysis) [11] can be used to detect rotation. Thresholding, edge detection or other image segmentation methods are used at first on the image. The mean vector and the covariance matrix are computed along with its eigenvector e. Two elements of eigenvector e_1 and e_2 helps to evaluate object rotation around the center.

In Phase-Only Correlation (POC) [12] et al. (or simply a "phase correlation") the rotation angle between two images is also estimated. First convert the image rotation into the image shift by polar mappings of the amplitude spectra of images, and then estimate the translational displacement between the polar mappings to obtain the rotation angle. Computational cost of 2D POC can be reduced using 1D. Here 1D POC functions for every pair of row lines in two polar mappings of two images are calculated, and then summarize all 1D POC functions to obtain rotational angle.

In [6], symmetry measure is used to detect the registration angle. Angle between symmetry axes of reference and sense images provide the actual value of angle of rotation. For symmetric images, this method takes a lot of computation time. This is because here symmetry axis SM_θ is calculated for each possible value of θ on the pair of reference and sense images. The angle of the symmetry axis in image f is defined as $\theta_m = Max_\theta(SM_\theta(f))$. From this point of view it is quite time hungry.

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In [13] different images symmetric or non-symmetric are registered using correlation with Nelder-mead simplex (NM) method [14] for function minimization. But the computational overhead of this method is very much dependent on the information inherent in the image. For images having narrow spectrum of gray level representation, this method takes more time to find the value at which the maximum correlation can be observed, as NM simplex method algorithm goes to expansion. In order to achieve high levels of accuracy, different reference samples are required. Moreover, it is devoid of independent functionality due to its inherent requirement of external intervention for the purpose of choosing threshold parameter.

In the present article, we use weighted linear regression technique to identify the rotational angle between the reference image and the sense image. Statistically line of regression offers the line of symmetry of an image. In our experiments, index of symmetry is chosen to provide more accurate result than symmetry measure [6]. Our method performs registration comparison at global level and is fully unsupervised being applicable on any type of image encompassing medical and non-medical. We apply our methodology to compare the performance with rotational registration [6], [13] algorithms based on symmetry measure.

The paper is organized as follows. Section 2 revisits concepts relevant to the area of rotation registration. The proposed procedure is described in details in Section 3. Section 4 presents registration results. Section 5 reports comparison with the other algorithm. Section 6 contains conclusion along with the discussion on the results.

2 Methodology

This section is devoted for quick recapitulation of the relevant concepts like rigid geometric transformation and line of regression.

2.1 Rigid Geometric Transformation

Rigid geometric transformation means mapping the pixel coordinates of the reference image into another coordinate system to form the sense image. Rigid transformation deals with transformation where shapes and angles are preserved under transformations like rotation, scaling and translation. Rotational transformation mainly preserves lengths (distance between two points), angles (angle between two lines) and area of the image. Mathematically, rotation by an angle θ is represented by

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \qquad (1)$$

where, (x,y) is a typical pixel coordinate in the original image, (x′,y′) is the coordinate in the rotated image.

Consecutive rotation means to rotate the image by an angle θ_1 and to rotate that rotated image further by an angle θ_2 and so on upto θ_n. These consecutive rotations on the image is same as composite rotation of the image at an angle ($\theta_1 + \theta_2 + \ldots + \theta_n$). This is because mathematicalty,

$$\begin{pmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2.2 Line of regression

The statistical linear regression model is

$$Y = \alpha + \beta x + \varepsilon \qquad (2)$$

where Y is the known dependent variable, α and β are unknown parameters subjected to statistical estimation, X is the known independent variable and ε is the error representing the component left unexplained by the above model [15].

The Sum of Square Error (SSE) in the quantitative measure of the error induced by a specific choice of α and β. The SSE is given by

$$SSE = (Y - \overline{\alpha + \beta x})^2 \qquad (3)$$

Incidentally the estimated α and β obtained by minimizing the SSE indicates the best linear regression fit of dependent variable Y on independent variable X in respect of the statistically minimum variance estimates of α and β.

3 Proposed Method

We now explain the procedure for the computation of axis of symmetry of both fixed(reference) and rotated(sense) images.

3.1 Foundation

Rotation is computed considering the centroid of an image as the origin. Let f(i,j) be the gray intensity value of an image of size M×N. The intensity weighted centroid (C_x,C_y) of the image is calculated.

To find the regression line of the object passing through the centroid, we calculate (i) the line passing through the centroid as well as (ii) find minimum weighted sum from all the image pixels suspended perpendicularly upon the line. Here weights are the corresponding pixel intensities. Thus we find

$$\min_{m,c} \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\frac{(y-\overline{mx+c})^2}{(1+m^2)} \qquad (4)$$

Let,

$$\phi = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\frac{(y-\overline{mx+c})^2}{(1+m^2)} \qquad (5)$$

Equation (5) is minimum at $(\hat{m},\hat{c})$ if $\partial\phi/\partial m = 0$ and $\partial\phi/\partial c = 0$ and the hessian matrix

$$\begin{pmatrix} \frac{\partial^2\phi}{\partial m^2} & \frac{\partial^2\phi}{\partial m\partial c} \\ \frac{\partial^2\phi}{\partial c\partial m} & \frac{\partial^2\phi}{\partial c^2} \end{pmatrix}$$

is positive definite at $(\hat{m},\hat{c})$.
From $\partial\phi/\partial m = 0$ and $\partial\phi/\partial c = 0$ at $(\hat{m},\hat{c})$ we have

$$\begin{pmatrix} \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)xy \\ \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)y \end{pmatrix} = B\begin{pmatrix} \hat{m} \\ \hat{c} \end{pmatrix} \qquad (7)$$

Or,

$$\begin{pmatrix} \hat{m} \\ \hat{c} \end{pmatrix} = (B)^{-1}\begin{pmatrix} \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)xy \\ \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)y \end{pmatrix} \qquad (8)$$

Where,

$$B = \begin{pmatrix} \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)x^2 & \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)x \\ \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)x & \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y) \end{pmatrix}$$

Basically, the axis of symmetry decomposes the image into two nonoverlapping regions F_1 and F_2 such that difference between the weighted sums of the two regions is minimum. i.e.,

$$\left| \sum_{(x,y)\in F_1} f(x,y) - \sum_{(x,y)\in F_2} f(x,y) \right| \quad \text{is minimum.}$$

This minimum value is called index of symmetry. Index of symmetry attaining value zero means perfect symmetry. For regular shaped object with uniform intensity distribution this value is zero.

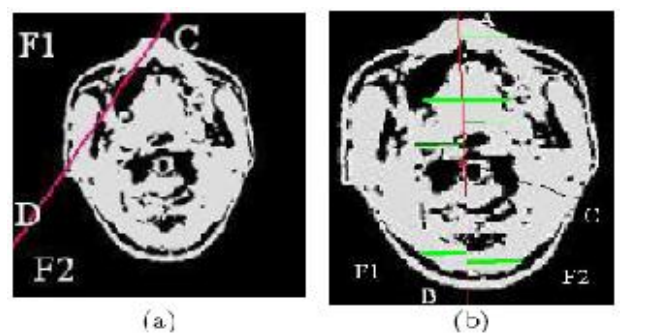


Fig. 1. (a) Brain image and any line CD that is not axis of symmetry of that image. (b) Brain image and axis of symmetry AB passing through centroid C.

In Fig. 1(b) C is the centroid. AB is the line of regression passing through C. It divides the image into two regions F_1 and F_2. For any pixel (x, y) ∈ F_1 or (x, y) ∈ F_2 we can draw perpendicular line on the line segment of regression AB. The difference between the sum of perpendicular distance on AB from each pixel (x, y) in the region F_1 and sum of perpendicular distance on AB from each pixel (x, y) in the region F_2 must be minimum. But in case of other arbitrary lines like CD in Fig. 1(a) the difference between weighted sum of two regions F_1 and F_2 may not be minimum.

The above idea can be presented in another way. Any arbitrary line $AX + BY + C = 0$ can be the axis of symmetry of any image if for any pair of image pixels (x,y) and (x',y'), (i) the difference between the image intensities at those coordinates is minimum and (ii) the difference between the perpendicular distance from those coordinates is minimum. Mathematically,
(i) f(x,y) - f(x',y') is minimum as well as
(ii) d(Ax + By + C) - d(Ax' + By' +C) is minimum,
where, (x,y), (x',y') are the coordinates of an image, f() is the intensity of the image at any given coordinate and d() is the perpendicular distance of any image pixel upon the line. Then the line AX + BY + C = 0 can be the axis of symmetry of that image. For any perfect symmetric object with uniform intensity distribution this difference ideally boils down to zero. Line of regression passing through centroid can be treated as the axis of symmetry of the image content.

It may be noted that there are regular geometric representations which may be perfect in the sense that they may have multiple axes of symmetry. For example, perfect circle, perfect squares etc. with uniform intensity distribution have infinitely many axis of symmetry. However, in real life situations the scope of encountering such perfect geometric structures is limited. In case of such an eventuality, our proposed method may be applied with respect to any choice of axis of symmetry out of various choices of axes of symmetry inherent in such structures. Accordingly, in such situations our proposed method will offer various alternative solutions.

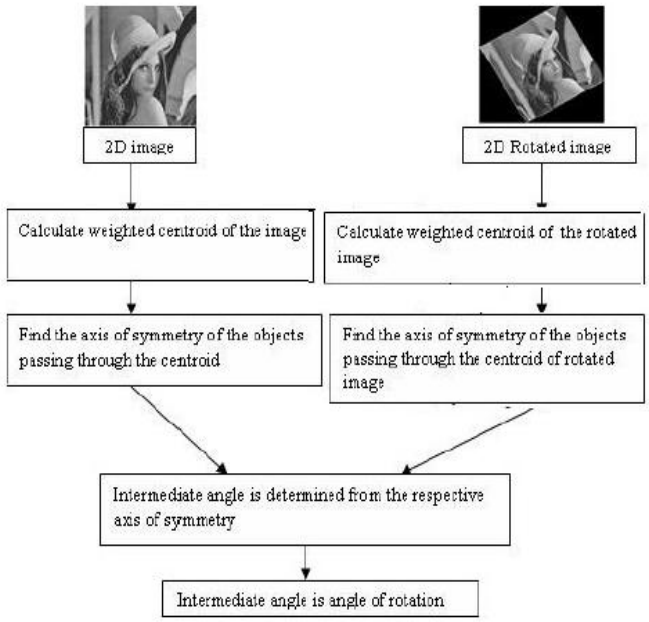


Fig. 2. Flowchart of the proposed method.

3.2 Our Algorithm

To solve the problem of rotational transformation, we compute the index of symmetry of the reference and sense images. So, the angular difference between the respective indices of symmetry of the reference image vis-a-vis the sense image, indicates the angle of rotation of the sense image with respect to the reference image. Using our method the indices of symmetry of the original image and the rotated versions of the image can be represented in table 1.

TABLE 1
EFFECT OF ANGULAR CHANGE ON AXIS OF SYMMETRY

Image	Index of Symmetry	Corresponding Angle	Angular difference between axes of symmetry between original and sensed image
Fig.3(a)	2.1962e+010	40	
Fig. 3(b)	2.1966e+010	60	60-40=20
Fig 3(c)	2.1962e+010	73	73-40=33
Fig. 4(a)	2.1963e+010	115	115-40=75
Fig. 4(b)	2.1965e+010	30	30-40=-10

In case of consecutive rotation, this method can be adopted for the cumulative amount of combined rotation with respect to the reference image. For example, if the sense image is produced using two consecutive rotation θ_1 and θ_2 respectively on the reference image (first θ_1 rotation followed by θ_2 rotation) then this method can easily identify the combined rotation ($\theta_1 + \theta_2$) to produce the sense image from the reference image. Figure 5(a) is the original image. This image is rotated at an angle of 40^0 as represented in fig. 5(b). Again this rotated version is re-rotated at an angle 30^0 further as shown in fig. 5(c). So, the reference image is rotated through an angle (40^0+30^0) i.e, 70^0 and the sense image is produced. Our proposed algorithm can easily identify this combined rotation as shown in the experiment section. For different types of rotation either single or consecutive our proposed method offers better result in terms of accuracy and time complexity than what reported in symmetry based method [6].

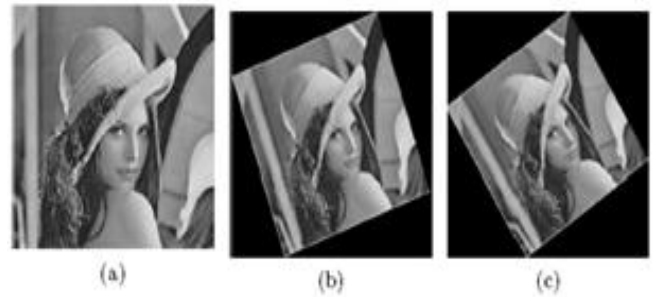


Fig. 3. (a) Lena image. (b) After 20^0 anticlockwise rotation of (a). (c) After 33^0 anticlockwise rotation of (a).

3.3 Limitation

The method proposed in this paper has the limitation that it can only be used to compute the rotational angle if the rotated version preserves information of original image.

4 EXPERIMENT

We carry out extensive experiments on our algorithm with different types of image inputs to confirm its performance. Some of them are reported here.

4.1 Experimental setup

The algorithm is implemented by using MATLAB 7.5.0 on a Core 2 Duo processor, 1.8 GHz, 2 GB RAM computer. Let the rotation angle is positive along anticlockwise direction.

4.2 Result

We applied our method on different types of images encompassing medical as well as non-medical applications such as remote sensor data, photography etc. Our method was applied on original image. Then the original has been rotated by a given angle using the bilinear interpolation method and our method of axis of symmetry has been identified. It appears from the observations that the axis of symmetry also undergoes a rotation by the similar amount in the figure. In other words, any rotation either simple or composite, induces

rotation of similar amount on the unique axis of symmetry of the image. This feature is being explored by our proposed algorithm. We take anticlockwise rotation as positive rotation. Figure 6 shows lena image being experimented with.

Fig. 4. (a) After 75^0 anticlockwise rotation of 3(a). (b) After 10^0 clockwise rotation of 3(a).

TABLE 2
PERFORMANCE OF OUR METHOD FOR DIFFERENT IMAGES WITH DIFFERENT ANGLE OF ROTATION

Original Image	Rotational Angle	Result using our mechanism
Lena (fig. 6(a))	30^0 (fig. 6(b))	$70-40=30^0$
Lena (fig. 6(a))	30^0 (fig. 7(a)) $+40^0$(fig. 7(b))	$110-40=70^0$
Baboon(fig. 8(a))	-20^0 (fig. 8(b))	$116-136=-20^0$
Brain (fig. 9(a))	3^0(fig. 9(b))	$5-2=3^0$

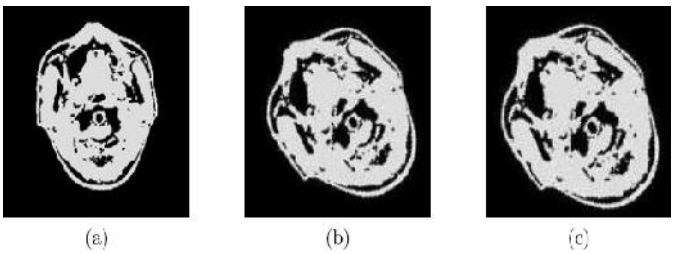


Fig. 5. (a)Brain image (b) After 40^0 anticlockwise Rotation of (a).(c) After 30^0 anticlockwise rotation of (b).

The size of the image is 217×218. Figures 6(a), 6(b) show original lena image and rotation after 30^0 respectively. In original image, line of regression makes an angle 40^0 with the vertical line passing through the centroid. In fig. 6(b) this line of regression makes an angle 70^0 with the vertical line passing through new centroid. Thus, the angle between these two lines is 30^0. So, it is observed that our method offers accurate result in this situation. Again fig. 7(a), 7(b) show consecutive rotations of the original image of lena (fig. 6(a)) by an angle 30^0 first (fig. 7(a)) followed by further by an angle 40^0 (fig. 7(b)).

Fig. 6. (a) Lena image. (b) After 30^0 anticlockwise rotation of (a).

Using our method, we find the angle as 70^0 (110^0 of rotated image - 40^0 of original image). Figure 8(a) shows baboon images of size 350×350. We rotate this image -20^0 (i.e., clockwise direction) which is represented in fig. 8(b). Using our method, we can find that the difference between two lines of regression is -20^0 indicating accurate result offered by our method. Next fig. 9(a) shows brain image with size 167×166 and rotated by 3^0 as available in fig. 9(b). This rotation is similarly captured by our method. The result of experiments are reported in table 2.

4.2 Complexity

Given an image of size MxN, the centroid of the image can be computed within O(MN) operations. Image rotation algorithm uses O(MN) operations to detect axis of symmetry of both reference and sense images.

Therefore overall complexity of the proposed algorithm is O(MN).

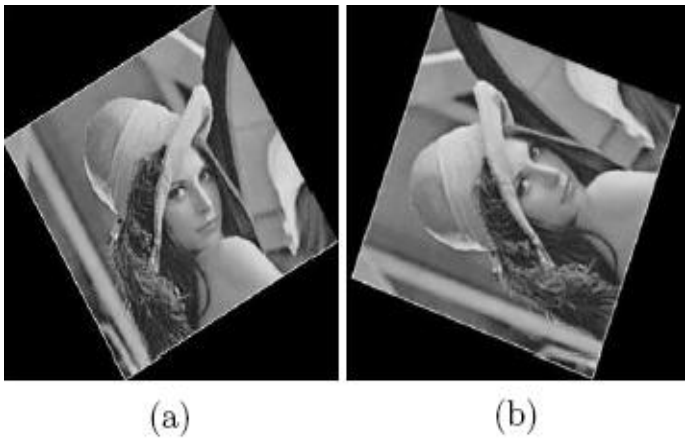


Fig. 7. (a) After 30^0 anticlockwise rotation of fig. 6(a). (b) After 40^0 anticlockwise Rotation of (a).

Algorithm 1 Rotation angle detection of a sense image S to a reference image R by computing weighted statistical regression

Ensure: Image S preserves the information of original image R

1: $M, N \Leftarrow Size(S)$

2: $M^{`}, N^{`} \Leftarrow Size(R)$

3: $in \Leftarrow \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} f_S(i,j)$

4: $X_S \Leftarrow \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} i f_S(i,j)$

5: $Y_S \Leftarrow \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} j f_S(i,j)$

6: $C_X \Leftarrow \frac{X_S}{in}$

7: $C_Y \Leftarrow \frac{Y_S}{in}$

8: Compute the weighted statistical regression line $Y = m_S X + c_S$ for image S passing through the centriod (C_X, C_Y) using equation (8)

9: Repeat steps 2 through 8 (with respect to) reference image R

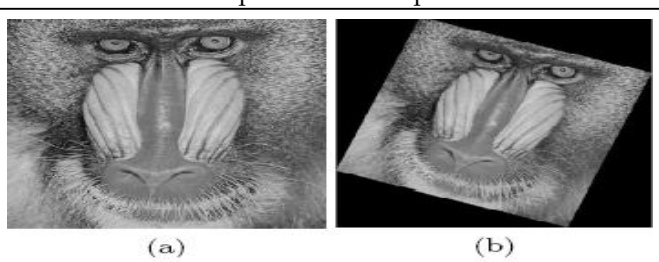
10: Find the angle θ between the two regression lines $Y = m_S X + c_S$ and $Y = m_R X + c_R$

11: Angle θ is reported as the angle of rotation

5 Comparison with other existing methods

The SM method [6] used symmetry measure for rotation registration. Here main idea is that the symmetry measure is at its maximum when the possible symmetry axis is the real symmetry axis. But this method takes $O(N^3)$ comparison to find the symmetry axis of any image of size NxN. In paper [13], NM simplex method is used for function minimization. But for images having narrow spectrum of gray level representation this method takes $O(N^3)$ computation.

Examples of observations in various images like fig. 10, 11, 12 are displayed. Table 3 indicates a comparison among NM simplex, SM(Symmetry Measure) and our proposed method under same experimental setup.



(a) (b)

Fig. 8. (a) Baboon image. (b) After 20^0 clockwise rotation of (a).

TABLE 3
Comparison with other existing methods

	Our method	SM	NM correlation
Fig. 10(a) rotated anticlockwise by 25^0 (Fig. 10(b))	204 - 179 = 25^0	20^0	25^0
Fig. 11(a) rotated anticlockwise by 38^0 (Fig. 11(b))	217 - 179 = 38^0	35^0	40^0
Fig. 12(a) rotated anticlockwise by 27^0 (Fig. 12(b))	207 - 180 = 27^0	24^0	Either 25^0 or 30^0
Fig. 12(b) again rotated anticlockwise by 5^0 (Fig. 12(c))	212 - 180 = 32^0	27^0	30^0

(a) (b)

Fig. 9. (a) Brain image. (b) After 3^0 anticlockwise Rotation of (a).

From table 3, it is clear that our proposed algorithm performs qualitatively better in comparison to other two methods namely SM, NM correlation for both symmetric and non-symmetric images. Here we append the histogram of the data obtained by respective methods and our method in fig. 13.

Nature of histograms indicate that the statistical dispersion of our method is convincingly smaller in comparison to other counterparts. Histograms of other two existing methods indicate multimodal tendency. It is difficult to use any known commonly used statistical distribution to represent the results generated by these algorithms. As a result, we have opted for non parametric statistical test for comparing these two techniques vis-a-vis ours.

We have three sets of data, obtained in the form of results by using the normalized distance measure of our algorithm, NM correlation and symmetry measure. Each data set contains 100 elements. Now, we apply Kolmogorov-Smirnov (K - S) two sample non-parametric test on two data sets X and Y for testing the equality of two unknown distributions yielding the respective data sets. This is because these two sets of data are coming from possibly two distinct unknown distribution functions F_X and F_Y respectively. Here we resort to nonparametric statistical test in order to avoid any specific bias that might crop up due to an assumption

regarding the nature of underlying distribution, the respective data sets are coming from. Depending on the nature of the alternative hypothesis, the test will offer, whether there is any stochastic ordering, among the random variables X and Y and, if there be any such ordering, which of the two is stochastically smaller than the other. $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_m$ are independent random samples from populations having distributions F_X and F_Y.

$$\text{Null hypothesis } H_0 : F_X(t) = F_Y(t)\ \forall t$$
$$\text{Alternative Hypothesis } H_1 : F_X(t) < F_Y(t)\ \forall t$$

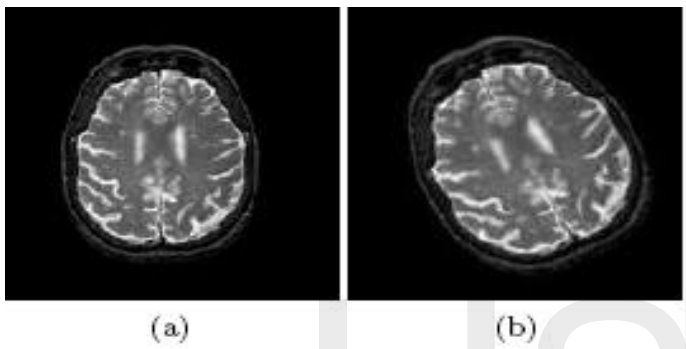


Fig. 10. (a) MR image. (b) After 25^0 anticlockwise Rotation of (a).

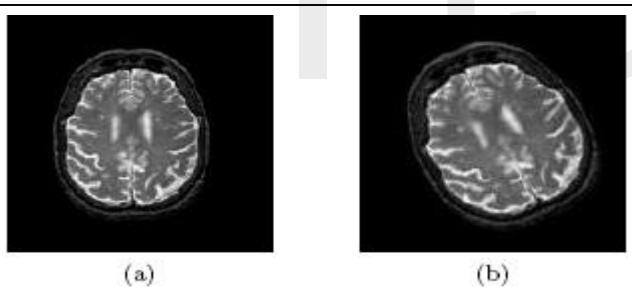


Fig. 11. (a) MR image. (b) After 38^0 anticlockwise Rotation of (a).

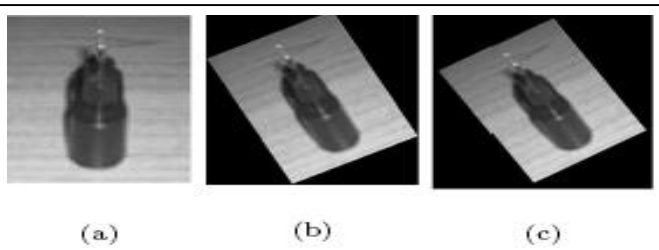


Fig. 12. (a) MR Original image. (b) After 27^0 anticlockwise Rotation of (a). (c) After 50^0 anticlockwise Rotation of (b).

We apply K-S test on data set 1 and data set 2. Maximum difference between the cumulative distributions D is 0.4158 i.e. $D^- = \max(F_2 - F_1) = 0.4158 > 0.1920$ (tabulated critical value). The test rejects the null hypothesis at the 5% significance level

TABLE 4
COMPARATIVE PERFORMANCE OF OUR ALGORITHM, NM CORRELATION AND SM METHOD

	Mean	Standard Deviation
Data Set 1 (Our Algorithm)	0.0396	0.3140
Data Set 2 (NM correlation)	0.2772	1.9700
Data Set 3 (SM Method)	1.2280	2.7700

i.e, this test accepts the alternative hypothesis that the data set 1 is stochastically smaller than data set 2. Again, we apply K-S test on data set 1 and data set 3. Maximum difference between the cumulative distributions D is 0.6238 i.e. $D^- = \max(F_3 - F_1) = 0.6238 > 0.1920$ (tabulated critical value). The test rejects the null hypothesis at the 5% significance level. In other words, this test accepts the alternative hypothesis with 95%

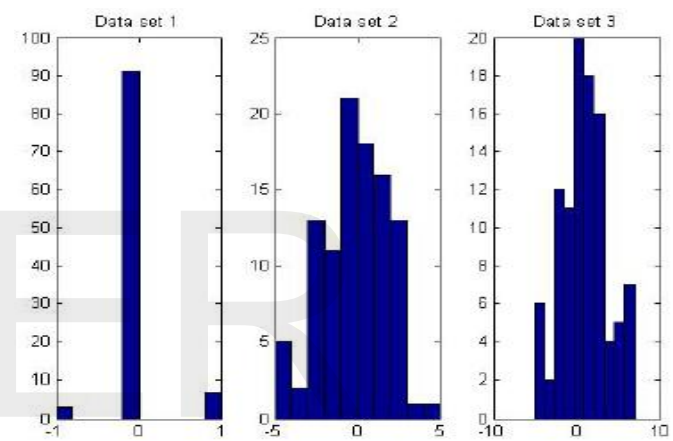


Fig. 13. Histogram of data sets

confidence that the first algorithm or data set 1 is stochastically smaller than data set 3 generated by SM method.

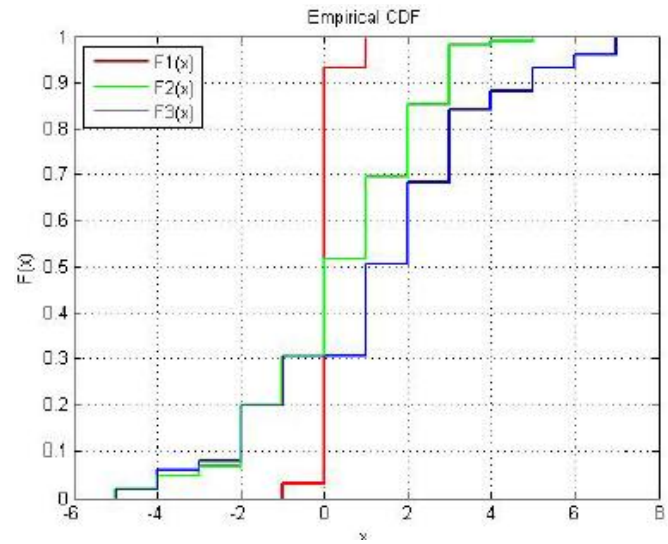


Fig. 14. Empirical cumulative distribution function of data set 1, 2 and 3.

6 Conclusion

This paper presents a novel technique to estimate two dimensional rotation in registering two different images of same object. The reference and sense images differing due to rotation or combination of rotations can be registered. The technique presented here can be applied on wide variety of image applications including medical or non-medical. This method is comparatively efficient as it only calculates the weighted linear regression line of both reference and sense images. This algorithm performs well for those images taken from same sensor, having same information. But for regulating images taken from different viewpoint or from different sensors or having different information content is beyond the scope of the proposed technique. However, the authors are presently engaged in that problem.

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